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We derive a finite-horizon version of the Shapiro-Stiglitz shirking model of unemployment.

Workers’ behavior may change as they approach the end of an employment contract.

Our model predicts that wages should be rising in age for an unchanged rate of unemployment.

A Non-perpetual Shirking Model

Yu-Fu Chen [a] and Gylfi Zoega [b, c*]

[a] Economic Studies, University of Dundee, 3 Perth Road, Dundee, UK
[b] Department of Economics, University of Iceland, 101 Reykjavik, Iceland
[c] Department of Economics, Birkbeck College, Malet Street, London, UK

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Abstract

We provide a finite-horizon counterpart to the Shapiro and Stiglitz model of unemployment to show how workers' effort falls as they approach the end of an employment spell. The model provides a reason for wages rising more rapidly than productivity.

Keywords: Wages, shirking, finite horizons, retirement.
JEL Classification: J31, J21

* Corresponding author: Gylfi Zoega, Department of Economics, University of Iceland, Sæmundargata, 101 Reykjavik, Iceland. Email: gz@hi.is; telephone: +354 525 5239; fax: + 354 552 6806. Financial support from the Icelandic Centre for Research and the University of Iceland Research Fund is gratefully acknowledged. We are grateful to an anonymous referee for comments.

1. Introduction

Every employment contract has a time dimension. There are workers on temporary contracts; workers who have been given an advance notice of dismissal know that their days on the job are numbered; and even workers who have safe permanent jobs realize that they will eventually retire. In this paper we extend and generalize the well-known model of wage setting by Shapiro and Stiglitz (S-S) (1984) to show how a worker's propensity to shirk his duties varies from the beginning to the end of an employment contract.

2. A non-perpetual model

We model a worker's effort decision when he has finite horizons leaving the infinite horizon case described in the S-S paper as a special case. There are three states of intertemporal utilities in the S-S model for workers with transitory probabilities to alternative states. These are the value of being employed, V_E (when not shirking) and V_S (when shirking), and the value of being unemployed, V_U. Workers receive the wage w when employed and unemployment benefits b_u when unemployed. Effort is exerted when employed workers are not shirking their duties while no effort is exerted when workers shirk. Workers discount future utility at rate ρ, face a constant probability of job termination b during the contract period and the probability q of being fired if caught shirking.

We start with a representative state i

$$V_i = \int_t^\infty u_i(s) e^{-\rho(s-t)} ds , \tag{1}$$

with transitory probability p_{ij} of moving to the alternative state V_j,where $u_i(s)$ is the immediate utility at time s for the state i. We can now introduce finite horizons by dividing the inter-temporal integral V_i into the periods of $t \leq time \leq T$ and $T \leq time \leq \infty$ where T denotes the time remaining until the end of horizon:

$$V_i = \int_t^\infty u_i(s) e^{-\rho(s-t)} ds = \int_t^T u_i(s) e^{-\rho(s-t)} ds + \int_T^\infty u_i(s) e^{-\rho(s-t)} ds . \tag{2}$$

The integral $\int_T^\infty u_i(s)e^{-\rho(s-t)}ds$ for time period $T \le time \le \infty$ can be rewritten as follows

$$\int_T^\infty u_i(s)e^{-\rho(s-t)}ds = e^{-\rho(T-t)}\int_T^\infty u_i(s)e^{-\rho(s-T)}ds\,. \tag{3}$$

Therefore, we need to discount the integral by the factor $e^{-p_{ij}(T-t)}$ if we would like to replace *T* with *t* since over the time period from *t* to *T*, the integral $\int_T^\infty u_i(s)e^{-\rho(s-T)}ds$ depreciates at the rate of p_{ij}:

$$e^{-\rho(T-t)}\int_T^\infty u_i(s)e^{-\rho(s-T)}ds = e^{-(\rho+p_{ij})(T-t)}\int_t^\infty u_i(s)e^{-\rho(s-t)}ds\,. \tag{4}$$

Equation (2) can now be rewritten as

$$V_i = \int_t^T u_i(s)e^{-\rho(s-t)}ds + e^{-(\rho+p_{ij})(T-t)}\int_t^\infty u_i(s)e^{-\rho(s-t)}ds = V_i^T + e^{-(\rho+p_{ij})(T-t)}V_i\,, \tag{5}$$

where $V_i^T = \int_t^T u_i(s)e^{-\rho(s-t)}ds$. Rearranging gives

$$V_i = \frac{V_i^T}{1-e^{-(\rho+p_{ij})(T-t)}}. \tag{6}$$

Equation (6) shows the relationship between the perpetual and non-perpetual intertemporal integrals for the state *i*. One can then apply equation (6) to three states: V_E, V_S, and V_U, with corresponding transitory probabilities: $p_{EU} = b$, $p_{SU} = b+q$, and $p_{UE} = a$, where $V_E^T = \int_t^T (w-\bar{e})e^{-\rho(s-t)}ds$ is the non-perpetual integral for the value of being a non-shirking employed worker who faces the probability *b* of moving to the unemployed state, $\bar{e}$ is the disutility of effort, $V_S^T = \int_t^T we^{-\rho(s-t)}ds$ is the non-perpetual integral for the value of being a shirking worker who faces the probability *b*+*q* of moving to the unemployment state and $V_U^T = \int_t^T b_u e^{-\rho(s-t)}ds$ is the non-perpetual integral for an unemployed worker who becomes employed with probability *a*, which denotes the probability of finding jobs.

We can derive the following three asset pricing equations by substituting equation (6) into the Bellman equations of the perpetual case of the S-S model;

$$\rho V_E^T = \left(w-\overline{e}\right)\left(1-e^{-(\rho+b)(T-t)}\right)+b\left(V_U^T \frac{1-e^{-(\rho+b)(T-t)}}{1-e^{-(\rho+a)(T-t)}}-V_E^T\right), \tag{7}$$

$$\rho V_S^T = w\left(1-e^{-(\rho+b+q)(T-t)}\right)+\left(b+q\right)\left(V_U^T \frac{1-e^{-(\rho+b+q)(T-t)}}{1-e^{-(\rho+a)(T-t)}}-V_S^T\right), \tag{8}$$

$$\rho V_U^T = b_u\left(1-e^{-(\rho+a)(T-t)}\right)+a\left(V_E^T \frac{1-e^{-(\rho+a)(T-t)}}{1-e^{-(\rho+b)(T-t)}}-V_U^T\right). \tag{9}$$

Using the no-shirking condition such that $V_E^T = V_S^T$ for equation (8) gives

$$\rho V_E^T = w\left(1-e^{-(\rho+b+q)(T-t)}\right)+\left(b+q\right)\left(V_U^T \frac{1-e^{-(\rho+b+q)(T-t)}}{1-e^{-(\rho+a)(T-t)}}-V_E^T\right). \tag{10}$$

There are three unknown variables, V_E^T , V_U^T , w, for (7), (9) and (10). Rearranging those three equations gives

$$\left(\rho+b\right)V_E^T - b\left(\frac{1-e^{-(\rho+b)(T-t)}}{1-e^{-(\rho+a)(T-t)}}\right)V_U^T - w\left(1-e^{-(\rho+b)(T-t)}\right) = -\overline{e}\left(1-e^{-(\rho+b)(T-t)}\right), \tag{11}$$

$$\left(\rho+b+q\right)V_E^T - \left(b+q\right)\left(\frac{1-e^{-(\rho+b+q)(T-t)}}{1-e^{-(\rho+a)(T-t)}}\right)V_U^T - w\left(1-e^{-(\rho+b+q)(T-t)}\right) = 0, \tag{12}$$

$$a\left(\frac{1-e^{-(\rho+a)(T-t)}}{1-e^{-(\rho+b)(T-t)}}\right)V_E^T - \left(\rho+a\right)V_U^T = -b_u\left(1-e^{-(\rho+a)(T-t)}\right). \tag{13}$$

Finally, using Cramer's rule gives the no-shirking condition for wages (see Appendix for details)

$$w = \frac{(1-B/A)\left[\overline{e}a\left(b+q\right)-b_u b\left(\rho+b+q\right)\right]+\left(B/A\right)b_u\rho q+\overline{e}\rho\left(a+b+\rho+q\right)}{(1-B/A)\left[\left(\rho+b\right)\left(\rho+a\right)+aq\right]+\rho q}, \tag{14}$$

where $A=\left(1-e^{-(\rho+b)(T-t)}\right)$ and $B=\left(1-e^{-(\rho+b+q)(T-t)}\right)$. Note that since $A < B$ we find that $(1 - B/A)$ is negative. The numerator of (14) falls faster than the denominator and the firm needs to pay wages that rise as the end of the contract period approaches. Because the effective discount rates for the shirking state is $\rho+b+q$ and higher than the effective discount rate for the non-shirking state $\rho + b$, shirking is less harmful to workers whose contract will expire soon.

For the perpetual case, we have *A*=*B*. Thus the no-shirking condition becomes

$$w = \frac{b_u \rho q + \bar{e}\rho\left(a+b+\rho+q\right)}{\rho q} = b_u + \bar{e} + \left(a+b+\rho\right)\frac{\bar{e}}{q} \;, \tag{15}$$

which is the original no-shirking condition of Shapiro and Stiglitz. Now denote the number of employed workers of age *t* by L_t. In steady state, the outflow from employment to unemployment equals bL_t and should equal to inflow of workers from unemployment to employment $a(N_t\text{-}L_t)$ where N_t is the number of workers of age *t* in the labor force.

$$bL_t = a(N_t - L_t)\,. \tag{16}$$

Thus $a+b = bL_t\left(N_t - L_t\right)^{-1} + b = bN_t\left(N_t - L_t\right)^{-1} = bN_t\left(N_t - L_t\right)^{-1} = b/u_t$ and we get $a = b\left(1-u_t\right)/u_t$. Substituting back into (14) gives the no-shirking condition in equilibrium as a relationship between wages and unemployment.

$$\begin{aligned} w = {} & \frac{(1-B/A)\left[\bar{e}b\left((1-u_t)/u_t\right)(b+q) - b_u b\left(\rho+b+q\right)\right]}{(1-B/A)\left[\left(\rho+b\right)\left(\rho+b\left((1-u_t)/u_t\right)\right)+b\left((1-u_t)/u_t\right)q\right]+\rho q} \\ & + \frac{(B/A)b_u\rho q + \bar{e}\rho\left(b/u_t+\rho+q\right)}{(1-B/A)\left[\left(\rho+b\right)\left(\rho+b\left((1-u_t)/u_t\right)\right)+b\left((1-u_t)/u_t\right)q\right]+\rho q}. \end{aligned} \tag{17}$$

It follows that each cohort of workers has a distinct wage curve – or no-shirking constraint – described by equation (17).

The non-shirking constraint is drawn in Figure 1 below as an upward-sloping non-shirking constraint for different age groups with benchmark values below the figure. There are only small differences between young and middle-aged workers. But the wage curves for older workers are substantially higher. It follows that the wage – or unemployment – needed to prevent a 40-year old worker from shirking his duties is not much higher than that needed to prevent a 20 year old worker from doing so but a significantly higher wage is needed to prevent a 50 year old worker from shirking than is the case of the 40 year old one.

As shown in Figure 2 we find that the wage required to prevent shirking rises rapidly in the 44-48 years age group when unemployment is 10%, in the 48-52 years age group when unemployment is 20% and in the 52-56 group when unemployment is a staggering 40%.

3. Discussion

Our model predicts that wages need to increase as a worker approaches the end of a contract when it is difficult to monitor effort and unemployment is held constant. It follows that wages are increasing in age for a given unemployment rate. The model describes wage setting in labor markets where there is substantial asymmetric information about workers' effort and monitoring is difficult, such as the market for professionals, managers, educated workers and also in large firms. In such markets older workers will either face a higher unemployment rate, which reduces the cost of employing them, be more productive due to experience or pushed into retirement.

We are not the first to propose an explanation for wages rising faster than productivity with age. Lazear (1979, 1981) derived a model where wages are set below productivity for young workers and then above productivity for workers approaching retirement for incentive reasons, hence also providing a justification for mandatory retirement. Another explanation for rising wage profiles is that of Frank and Hutchens (1993) who assume that satisfaction depends on the rate of change of consumption. Rising wage profiles may be desired by workers who find it difficult to postpone consumption through voluntary savings.

While rising wage profiles may provide incentives and be desired by the workforce our model provides additional insights into the relationship between time until retirement and productivity. There is evidence that the behavior of workers may change as they approach the end of tenure. Figlio (1995) found that the decision to retire from politics results in political shirking – meaning less party discipline – using a multi-year panel data set. Tien (2001) also found that voluntarily retiring members of Congress shirked by failing to represent the interests of their constituents. Parker and Powers (2002) found that spending on foreign travel is higher among members of Congress who are about to leave office. DeBacker (2012) detects shirking by senators in their last term that is limited by political parties that constrain the politician depending on his post-Senate career choices. In sports, Krautmann and Solow (2009) examined incentives in baseball contracts and found that players who are less likely to sign a subsequent contract showed worse performance.

There are empirical studies that support the predictions of our model. Medoff and Abraham (1981) find an association between experience and relative earning but no association between experience and relative performance for managers and professionals in two large U.S. companies. Dostie (2006) use Canadian data and finds that productivity is lower than wages for older workers who have at least an undergraduate degree although the evidence on the direct effect of age on productivity is mixed. Lallemand and Ryck (2009) attribute low employment rates among older workers in Belgium to older workers being more costly to employ and less productive than prime age workers. Studying productivity data from large Belgian firms they find that a higher share of older workers lowers average productivity. In contrast, Börsch-Supan and Weiss (2008) study productivity in a German car manufacturing company and find that experience keeps the productivity of older workers from falling by giving them an ability to avoid making serious errors.

4. Conclusions

By extending the model of Shapiro and Stiglitz (1984) we have found that workers tendency to shirk their duties increases as they approach the end of tenure. Moreover, the threat of unemployment has a smaller effect on these workers requiring firms to raise their wages or make them redundant.

References

Börsch-Supan, Axel H. and Matthias Weiss (2008), "Productivity and the Age Composition of Work Teams: Evidence from the Assembly Line," MEA Discussion Paper No. 148-07. Available at SSRN: http://ssrn.com/abstract=1335390 or http://dx.doi.org/10.2139/ssrn.1335390.

DeBacker, Jason (2012), "Political Parties and Political Shirking," *Public Choice*, 150 (3-4), 651-670.

Dostie, B. (2006), "Wages, Productivity and Aging," *De Economist*, 159, 139-158.

Figlio, David N. (1995), "The Effect of Retirement On Political Shirking: Evidence From Congressional Voting," *Public Finance Review*, 23 (2), 226-241.

Frank, Robert H. and Robert M. Hutchens (1993), "Wages, Seniority, and the Demand for Rising Consumption Profiles," *Journal of Economic Behavior & Organization*, 21 (3), 251-276.

Krautmann, Anthony C. and John L. Solowhttp://jse.sagepub.com/content/10/1/6.short - aff-2 (2009), "The Dynamics of Performance Over the Duration of Major League Baseball Long-Term Contracts," *Journal of Sports Economics*, 10 (1), 6-22.

Lallemand, T. and F. Ryck (2009), "Are Older Workers Harmful for Firm Productivity?", *De Economist*, 157, pp. 273–292.

Lazear, Edward P. (1979), "Why Is There Mandatory Retirement?" *The Journal of Political Economy*, 87 (6), 1261-1284.

Lazear, Edward P. (1981), "Agency, Earnings Profiles, Productivity, and Hours Restrictions," *American Economic Review*, 71, 606-620.

Medoff, James L. and Katharine G. Abraham (1980), "Experience, Performance, and Earnings," *The Quarterly Journal of Economics*, 95 (4), 703-736.

Parker, Glenn R. and Stephen C. Powers (2002), "Searching for Symptoms of Political Shirking: Congressional Foreign Travel," *Public Choice*, 110 (1-2), 173-191).

Shapiro, Carl and Joseph E. Stiglitz (1984), "Equilibrium Unemployment as a Worker Discipline Device," *American Economic Review*, 74 (3), 433-44.

Tien, Charles (2001), "Representation, Voluntary Retirement, and Shirking in the Last Term," *Public Choice*, 106 (1-2), 117-130.

Appendix

Equations (11)-(13) can be written as follows:

(A1) $$(\rho+b)V_E^T - b\frac{A}{C}V_U^T - Aw = -A\bar{e},$$

(A2) $$(\rho+b+q)V_E^T - (b+q)\frac{B}{C}V_U^T - Bw = 0,$$

(A3) $$a\frac{C}{A}V_E^T - (\rho+a)V_U^T = -Cb_u,$$

where $A=\left(1-e^{-(\rho+b)(T-t)}\right)$, $B=\left(1-e^{-(\rho+b+q)(T-t)}\right)$, and $C=\left(1-e^{-(\rho+a)(T-t)}\right)$.

Cramer's rule gives the solutions for no-shirking conditions wages

(A4) $$w=\frac{\begin{vmatrix}(\rho+b) & -b\frac{A}{C} & -A\bar{e}\\ (\rho+b+q) & -(b+q)\frac{B}{C} & 0\\ a\frac{C}{A} & -(\rho+a) & -Cb_u\end{vmatrix}}{\begin{vmatrix}(\rho+b) & -b\frac{A}{C} & -A\\ (\rho+b+q) & -(b+q)\frac{B}{C} & -B\\ a\frac{C}{A} & -(\rho+a) & 0\end{vmatrix}}.$$

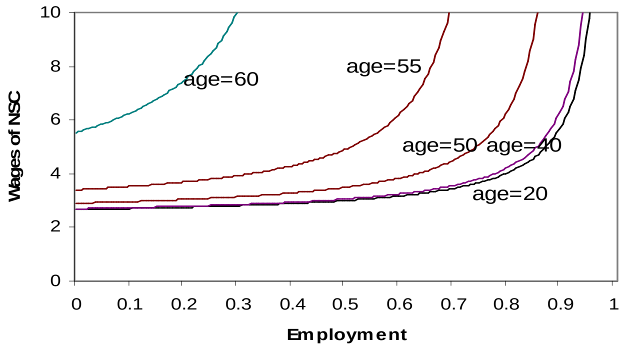
Expanding the determinants gives

(A5) $$w=\frac{Bb_u(\rho+b)(b+q)+A\bar{e}(\rho+b+q)(\rho+a)-Ba\bar{e}(b+q)-Abb_u(\rho+b+q)}{A(\rho+b+q)(\rho+a)+Bab-Ba(b+q)-B(\rho+b)(\rho+a)}.$$

Equation (A5) can further be simplified as follows,

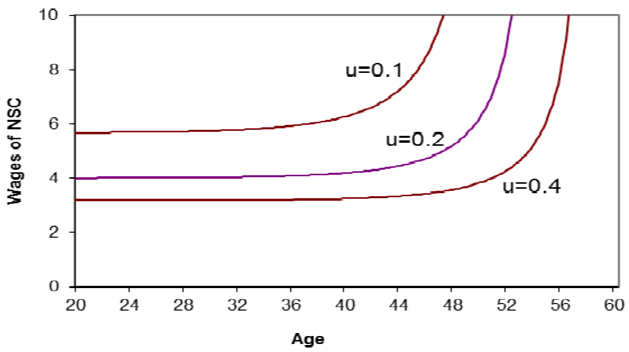
(A6) $$\begin{aligned} w &= \frac{(B-A)b_u b(\rho+b+q)+Bb_u\rho q+(A-B)\bar{e}a(b+q)+A\bar{e}\rho(a+b+\rho+q)}{(A-B)\left[(\rho+b)(\rho+a)+aq\right]+A\rho q}\\ &= \frac{(A-B)\left[\bar{e}a(b+q)-b_u b(\rho+b+q)\right]+Bb_u\rho q+A\bar{e}\rho(a+b+\rho+q)}{(A-B)\left[(\rho+b)(\rho+a)+aq\right]+A\rho q}\\ &= \frac{(1-B/A)\left[\bar{e}a(b+q)-b_u b(\rho+b+q)\right]+(B/A)b_u\rho q+\bar{e}\rho(a+b+\rho+q)}{(1-B/A)\left[(\rho+b)(\rho+a)+aq\right]+\rho q}. \end{aligned}$$

Figure 1. Age-dependent wage curves



Parameter values: ρ = 0.1, b = 0.1, q = 0.3, e = 1.0, b_u = 1, N = 1000. Note that T=45 implies that age = 20; and T=5 means that age = 60, if we assume that age 65 is the age at which workers are no longer willing to work.

Figure 2. The non-shirking wage and age



Parameter values: Same as in Figure 1.